

## Credit Cards and Student Loans

Credit cards and student loans are compounded continuously so you can compute the amount you owe using the following formula:

$$A = P e^{rt}$$

where P is the principal or the amount you borrowed, t is the time in years that has past, r is the annual interest rate written as a decimal so that 15 percent is .15 and 4 percent is .04, and A is the amount you owe now.

**Example I:** You are at a department store and have just made a one hundred dollar purchase. The clerk says you can have a store credit card and you don't have to pay minimum payments for 2 years. You agree and sign the forms which mention the credit card charges 17 percent interest. How much do you owe in 2 years?

**Solution:** Since you aren't making any payments for two years, you simply figure out how much the original one hundred dollars owed increases over two years using:

$$A = P e^{rt} \text{ where } r=.17 \text{ and } t=2.$$

So you owe:

$$A = (100) e^{(.17)^2} = 100 e^{(.34)} = 100 (1.404948) = 140.49.$$

That is one hundred and forty dollars and forty nine cents for only one hundred dollars worth of stuff!!!!

**Example II:**

Next time you go to the store and make a one hundred dollar purchase the clerk makes the same offer and when you turn it down, the clerk says that you will get a 10 percent discount on the day's purchase as well. So you agree, get the 10 percent discount so your charge is only ninety dollars, and sign the forms which say it is 17 percent interest. How much do you owe in 2 years?

**Solution:**

This is the same as last time except now your principal, P, is only ninety dollars:

So you owe:

$$A = (90) e^{(.17)^2} = 90 e^{(.34)} = 90 (1.404948) = 126.45$$

which is one hundred twenty six dollars and 45 cents for only one hundred dollars worth of stuff!

**Problems:**

**Be careful plugging in the information into your calculator.**

**Make sure you can work out the two examples using your calculator.**

**Problem 1:**

Suppose you buy four hundred fifty dollars worth of textbooks at the bookstore with a credit card that charges 16 percent interest and requires no minimum payments for one year. How much do you owe at the end of the year?

**Problem 2:**

Suppose you take out a student loan for 15,000 dollars tuition at NYU. The college loan charges only 8 percent interest and you don't need to pay a thing until you finish your studies. How much do you owe after 4 years? How much do you owe after five years if that's how long it takes you to finish college? What if you finish college in only four years but spend another seven getting a doctorate?

**Problem 3:**

Suppose you take out a one thousand dollar loan for tuition at CUNY on September 1 and the loan charges 9 percent interest. In December you win a scholarship with a two thousand dollar stipend and decide to pay off the loan on Jan 1. How much do you owe?

**Problem 4:**

Suppose you take out a one thousand dollar loan for tuition at CUNY that charges 7 percent interest to take a JAVA programming course. You get a paid internship the following summer programming for a finance company and decide to pay off the loan in only 10 months. How much do you owe?

**Problem 5:**

You have a wealthy father who promises to pay five thousand dollars in tuition for you. When he doesn't give you the check you use your credit card which charges 18 percent interest. You finally get the five thousand in the mail 45 days later and pay off the bill. How much more do you owe after paying the 5000 dollars?

**Problem 6:**

You make a purchase of 500 dollars at a department store and get a 15 percent discount for using a new credit card with 17.5 percent interest and no payments due for a year. How much do you owe in 6 months?

**Problem 7:**

Suppose you buy two hundred fifty dollars worth of textbooks at the bookstore on February 1 with a credit card that charges 16 percent interest and requires no minimum payments for one year. How much do you owe July 1?

**Problem 8:**

Suppose you take out a two thousand dollar loan for tuition at CUNY that charges 6 percent interest. You graduate in only three years and pay off the loan after working 2 months. How much do you pay?

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# Inverting Functions with Exponentials and Logs

Here we extend techniques using inverse of a function with exponentials and logs:

## **Example:**

Let

$$f(x) = e^{3x-1}$$

What are the domain and range of  $f$ ?

What is the inverse of  $f$ ?

Be sure to state the domain and range of the inverse as well as giving us the formula.

## **Solution:**

Well the domain of  $f$  is  $(-\infty, \infty)$  because there is an output for every possible input.

The range is  $(0, \infty)$  because all the outputs are positive.

To get a formula for  $f^{-1}(x)$ , we write  $y = e^{3x-1}$  and solve for  $x$ .

$$y = e^{3x-1}$$

$$\ln(y) = \ln(e^{3x-1})$$

$$\ln(y) = 3x - 1$$

$$\ln(y) + 1 = 3x$$

$$(\ln(y) + 1)/3 = x$$

We have solved for  $x$ , which is the output of  $f^{-1}$  when the input is  $y$ . So replace  $x$  by  $f^{-1}(x)$  and replace  $y$  by  $x$ :

$$f^{-1}(x) = (\ln(x) + 1)/3.$$

Note this is NOT the same as  $\ln(x + 1)/3$  or  $\ln((x + 1)/3)$ .

The domain of  $f^{-1}$  is the range of  $f$ , which is  $(0, \infty)$  and that makes sense because we can take  $\ln(x)$  for any positive number  $x$ .

The range of  $f^{-1}$  is the domain of  $f$ , is  $(-\infty, \infty)$ .

$$f(f^{-1}(x)) = f((\ln(x) + 1)/3) \text{ plugging in the formula for } f^{-1}(x) \quad (1)$$

$$= e^{3((\ln(x)+1)/3)-1} \text{ substituting into the formula for } f \quad (2)$$

$$= e^{(\ln(x)+1)-1} \text{ canceling the threes} \quad (3)$$

$$= e^{\ln(x)} \text{ canceling the ones} \quad (4)$$

$$= x \text{ because } \ln \text{ is the inverse of } e^x. \quad (5)$$

Also check:

$$f^{-1}(f(x)) = f^{-1}(e^{3x-1}) \text{ plugging in the formula for } f(x) \quad (6)$$

$$= (\ln(e^{3x-1}) + 1)/3 \text{ substituting into the formula for } f^{-1} \quad (7)$$

$$= ((3x - 1) + 1)/3 \text{ because } \ln \text{ and } e \text{ cancel since they are inverses} \quad (8)$$

$$= (3x)/3 = x \quad (9)$$

Since:

$f^{-1}(f(x)) = x$  and  $f(f^{-1}(x)) = x$  we have verified that we have the correct formula for the inverse.

1. Find the domain, range and inverse for  $f(x) = e^{2x+7}$ .

2. Find the domain, range and inverse of  $f(x) = \ln(x-5)$ .  
Warning, the  $\ln$  is only defined on positive numbers so one needs  $x - 5 > 0$ .

3. Find the domain, range and inverse of  $f(x) = 5^{x+2}$ .

4. Find the domain, range and inverse of  $f(x) = \text{Log}_2(8x)$ .

5. Find the domain, range and inverse of  $f(x) = e^x + 3$ . Be careful with the range.

Think about how graphs shift.

6. Find the domain, range and inverse for  $f(x) = 2^{4x-5}$ .

7. Find the domain, range and inverse of  $f(x) = \ln(x + 13)$ .

8. Find the domain, range and inverse of  $f(x) = 2^{x+5}$ .

9. Find the domain, range and inverse of  $f(x) = \log_5(25x)$ .

10. Find the domain, range and inverse of  $f(x) = e^{2x} - 10$ .